A Tabu Search Algorithm for Unsplittable Capacitated Network Design

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Abstract: The Multicommodity Capacitated Network Design (MCND) problems with splittable flow variables are NP-hard, and with binary flow variables or unsplittable MCND, the complexity of the problems are increased significantly. Metaheuristics are the solution methods that can be developed to solve these problems efficiently. This paper presents a Tabu Search approach for unsplittable MCND problem. In the proposed solution method, an innovative representation and neighborhood structure are presented.

Keywords: Multicommodity Capacitated Network Design problem, unsplittable MCND, Metaheuristic Methods, Tabu Search Algorithm.

1. Introduction

The network design problem is one of the important problems in combinatorial optimization that have numerous applications in various fields such as transportation [1], telecommunication [2], and distribution planning [3]. One type of the network design models is splittable Multicommodity Capacitated Network Design (MCND) Problem. In this model, multiple commodities must be routed between different points of origin and destination on the arcs with limited capacity. The MCND problem seeks a network with minimum cost which satisfies demands of commodities. This minimum cost is the sum of the fixed and variable cost. In contrast with splittable MCND, flows of commodities cannot be separated on different routes in unsplittable MCND [1].

There are effective exact solution methods for uncapacitated network design problem, but adding capacity to the arcs of network design problem adds more complexity to this problem. There are theoretical and empirical evidence that the capacitated network design problems are NP-hard [1]. As a result, the researchers proposed approximate methods (Heuristics and metaheuristics) to solve them [4]-[8].

This paper proposes a Tabu Search (TS) [9] metaheuristic algorithm as a metaheuristic solution method for unsplittable MCND problem based on an innovative representation for representing solutions. In the next section, unsplittable MCNDP optimization model is presented. In section 3, the proposed TS solution method is described.

2. Unsplittable MCND Problem

Suppose a directed network $G = (N, A)$, in which N is the set of nodes and A is the set of directed arcs. Let K be the set of commodities, then the amount of each commodity k which must flow from its Origin $O(k)$ to its destination $D(k)$ is d_k. Let c_{ij}^k and f_{ij} be the per unit arc routing cost of commodity k on arc (i, j), and fixed arc designing cost of arc (i, j), respectively. The unsplittable MCND problem can be formulated as follows:

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k x_{ij}^k + \sum_{(i,j) \in A} f_{ij}\, y_{ij} \tag{1}$$

subject to

$$\sum_{j \in N} x_{ij}^k - \sum_{l \in N} x_{li}^k = \begin{cases} 1 & if \quad i = O(k) \\ -1 & if \quad i = D(k) \quad for\ all \quad K \in k \\ 0 & otherwise \end{cases} \tag{2}$$

$$\sum_{k \in K} x_{ij}^k d_k \le u_{ij} y_{ij} \qquad for\ all\ \ (i, j) \in A \tag{3}$$

$$x_{ij}^k \ , \ \ y_{ij} = 0 \ or\ 1 \qquad for\ all\ (i, j) \in A, \ \ k \in K \tag{4}$$

Where, y_{ij} and x_{ij}^k are decision variables. y_{ij} is 0 if arc (i, j) is closed, and 1 if it is open. x_{ij}^k is 1 if commodity k is shipped on arc (i, j), otherwise is 0. The objective function (1), is to minimize variable and fixed costs. Constraints (2) are the usual balancing equations of network flow problem. Constraints (3) demonstrate that the sum of flows on each arc (i, j) must not exceed the capacity u_{ij} of the arc.

3. The Proposed TS for the Unsplittable MCND

For the proposed TS method for unsplittable MCND problem, we inspired representation from [10] in which a representation for genetic algorithm for shortest path problem was proposed. To illustrate the proposed neighborhood structure, suppose figure 1 as a partial network of unsplittable MCND problem with 3 commodities. The capacity of each arc has been shown on it. Table 1 shows origin, destination and demand of each commodity k. We use a $m \times n$ matrix to represent a solution,